Let’s play a game

- Two-player, simultaneous, one-shot game (money request game)
- Game: players announce a number between 11 and 20

Payoff Structure:

- Players always receive the number of tokens they announce
- If player announces one less token than their opponent, then they receive X={20,80} tokens
- If both players announce the same number, then they both receive 10 tokens

- **What would you play?**

Endogenous Depth of Reasoning

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Review of Economic Studies (2016)

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Motivation

Introduction

- Existing behavioural game theoretical models fail to make good predictions in strategic games

- Why? Behaviour =/= player sophistication (endogeneity problem)
  - Behaviour is a function of the player's cognitive abilities, incentives and beliefs about the opponent's reasoning process

- Based on level-k reasoning and theory of mind, but player's "depth of reasoning" (DoR, cognitive bound/level of sophistication) is **endogenously** determined

Side note: Vincent Crawford suggests that people can only reason up to k=4 (EBE summer school, 2024)

Theoretical Model

- Assumption = player's reasoning process is step-wise
- Player's cognitive bound is endogenously determined using a cost-benefit analysis

Player's DoR = f(game payoffs, player's cognitive abilities)

  - *Benefit: game payoffs = incentive to reason*
  - *Cost: player's cognitive abilities = level of sophistication*
  - Higher payoff -> more incentive to reason -> higher DoR

- Player's behaviour depends on:
  - Player's DoR (cognitive bounds)
  - Player's belief about the opponent's sophistication
  - Player's belief of the opponent's belief (about their sophistication), and other higher-order beliefs

Modified 11-20 game

- Two-player, simultaneous, one-shot game (money request game)
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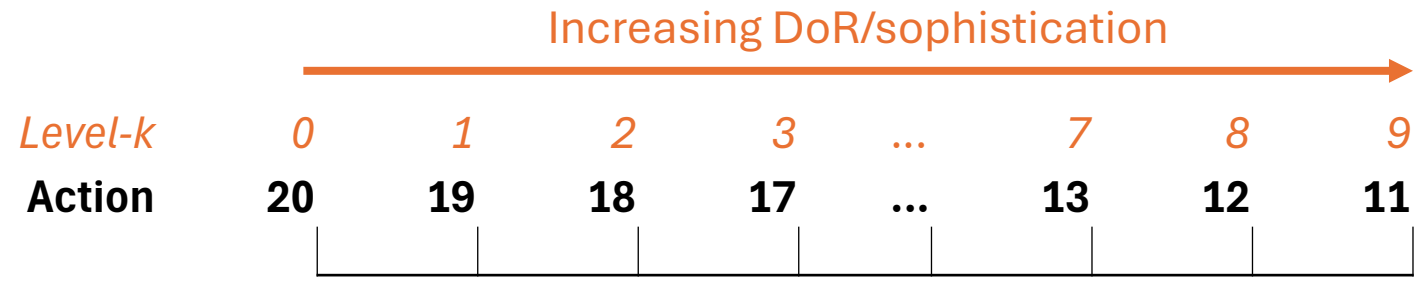
Modified 11-20 game

Possible strategies:

- Naïve player (Level-0 player) = choose 20
- Level-1 player = choose 19
- Level-k player = choose 20-k *(best response to level-k-1 opponent)*

- Optimal strategy = choose 11
    - Payoff = 21

Example



- Suppose you are a level-3 player (i.e., your DoR is 3)
- You know you can play 17

- If you believe your opponent is a level-3 or above player, then your best response given your max. level of DoR is to play 17

- If you believe your opponent is less sophisticated than you, then you play according to your belief of your opponent's DoR
  - E.g., if you believe your opponent uses level-1 reasoning (and plays 19), then your best response is to play 18, NOT 17

Hypotheses

1. **Greater incentives -> greater DoR**

2. **Change in belief (about the opponent) -> change in behaviour**
   - If the player believes that the opponent is more sophisticated, they will increase their level of play up to their cognitive bound

3. **Change in higher-order beliefs (about the opponent's belief in their sophistication) –> change in behaviour**
   - If the player believes that the opponent thinks they are more sophisticated, they will increase their level of play up to their cognitive bound
   - If player believes that the opponent thinks they are is less sophisticated, then they will play a less sophisticated strategy

Experimental Design

TABLE 1

Treatment summary: label I refers to "math and sciences" or to "high" subjects, and label II refers to "humanities" or to "low" subjects

Treatment	Own label	Opponent's label	Own payoffs	Opponent's payoffs	Replacement of opponent's opponent
Homogeneous [A]	*I* (*II*)	*I* (*II*)	Low	Low	No
Heterogeneous [B]	*I* (*II*)	*II* (*I*)	Low	Low	No
Replacement [C]	*I* (*II*)	*II* (*I*)	Low	Low	Yes
Homogeneous-high [A+]	*I* (*II*)	*I* (*II*)	High	High	No
Heterogeneous-high [B+]	*I* (*II*)	*II* (*I*)	High	High	No
Replacement-high [C+]	*I* (*II*)	*II* (*I*)	High	High	Yes

Notes: There are 120 subjects for each treatment (sixty subjects for each classification).

Exogenous vs. endogenous classification/labelling

Exogenous labelling: "maths and sciences" vs. "humanities"

Endogenous labelling: based on cognitive test scores

Experimental Design

Low vs. high incentives = bonus 20 vs. 80 if player choses a number exactly one below their opponent's number

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Replacement [C]: higher-order belief testing

"Two students of a particular type play against each other. You play against the number that one of them has picked"

Data Analysis

1. Greater incentives -> greater DoR
   - A, B, C vs. A+, B+, C+

2. Change in belief (about the opponent) -> change in behaviour
   - A vs. B, A+ vs. B+

3. Change in higher-order beliefs –> change in behaviour
   - B vs. C, B+ vs. C+

Treatment	Own label	Opponent's label
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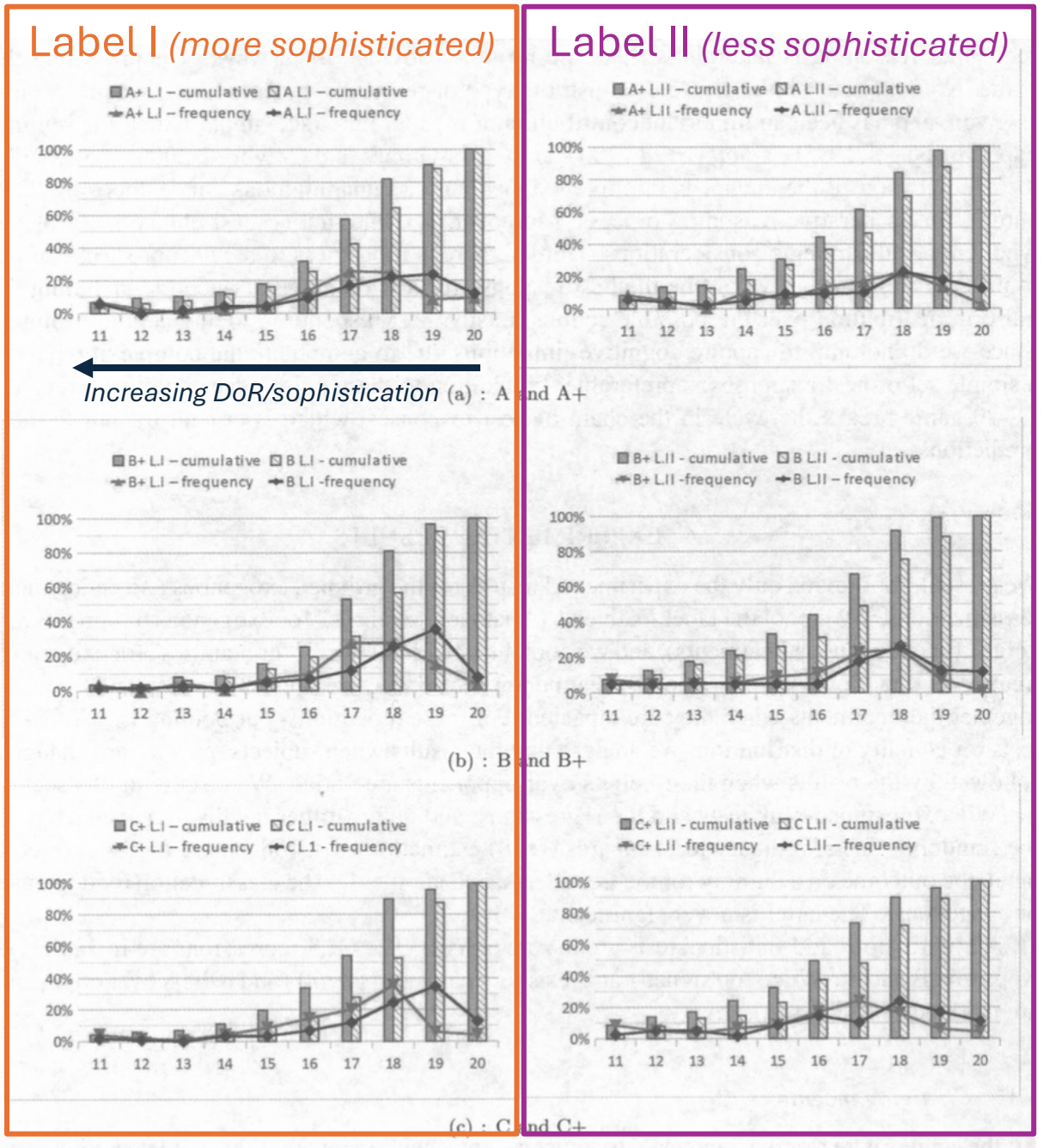
Key findings (1/2)

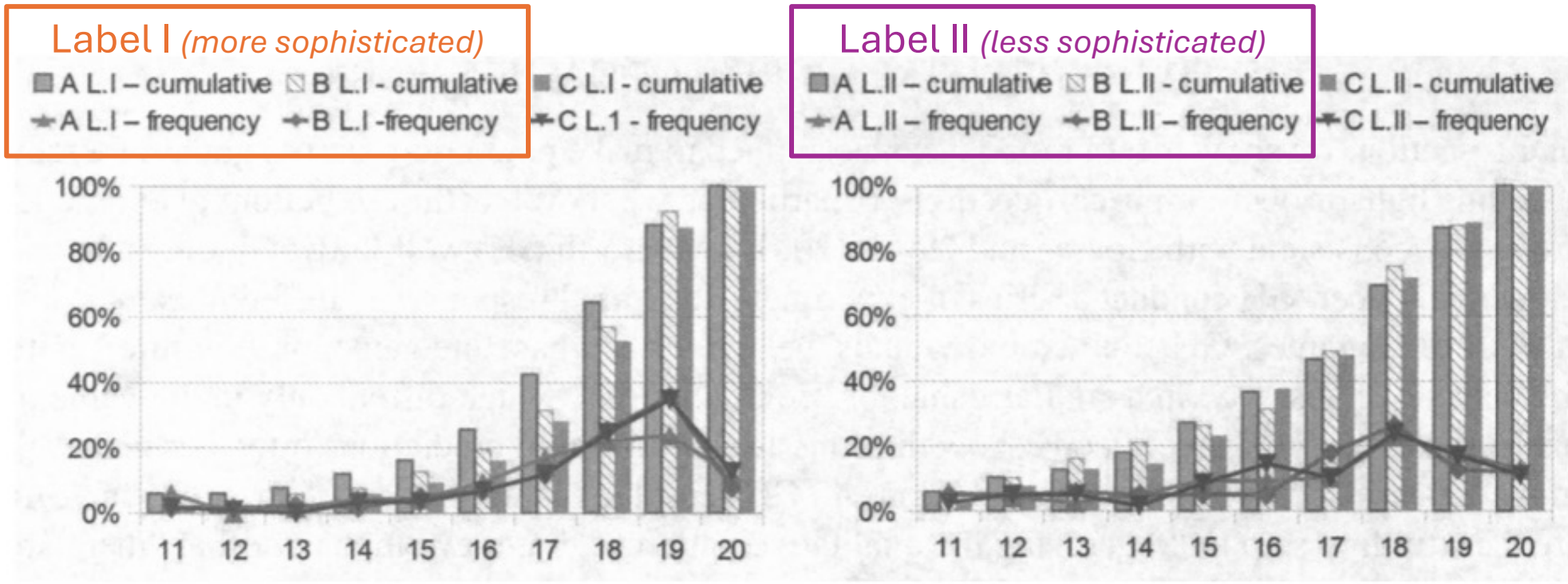
1. **Greater incentives -> greater DoR**
   - Participants chose lower numbers (more sophisticated level of play) when incentives were higher



Key findings

2. Change in belief (about the opponent) -> change in behaviour

- Label I (II) chose lower (higher) levels of play when playing against Label II (I) compared to Label I (II)

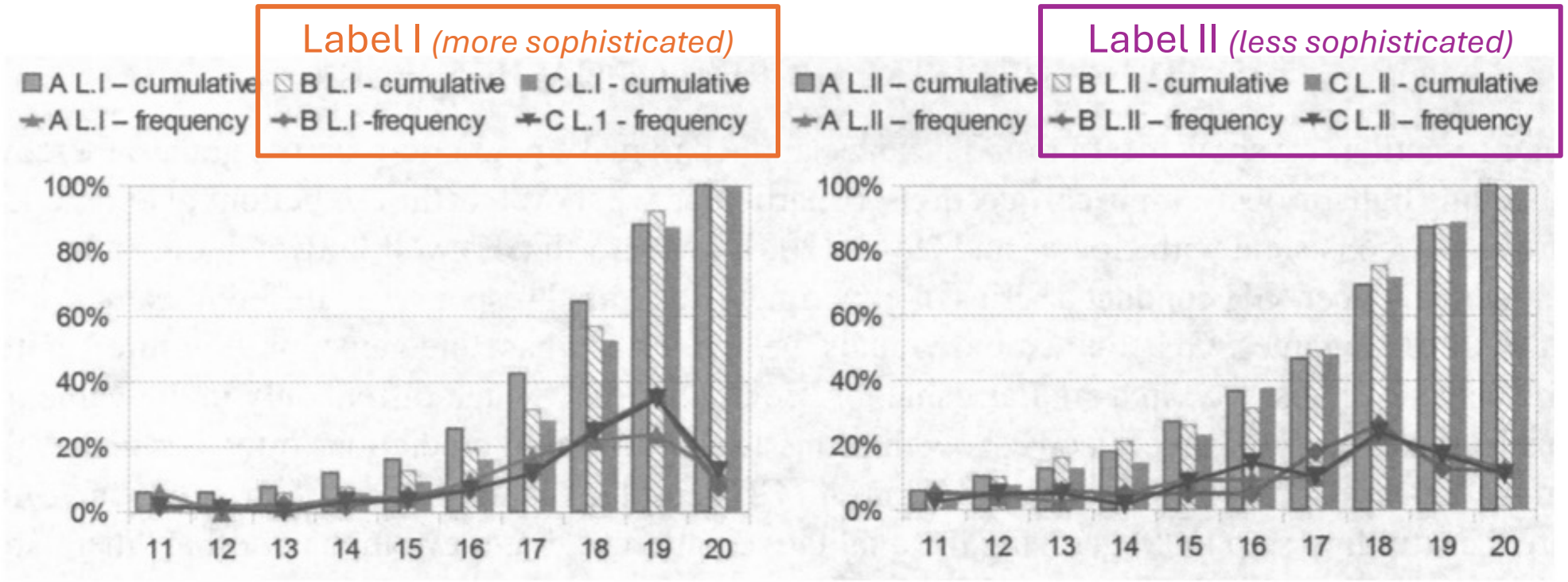


Key findings

Replacement [C]: higher-order belief testing
"Two students of a particular type play against each other. You play against the number that one of them has picked"

3. Change in higher-order beliefs –> change in behaviour

- Label I (II) chose even lower (higher) levels of play when playing against a game with Label IIs (Is)



Conclusion

- Behaviour =/= player sophistication (endogeneity problem)

- Player's behaviour depends on:
  - Player's DoR (cognitive bounds)
  - Player's belief about the opponent's sophistication
  - Player's belief of the opponent's belief (about their sophistication), and other higher-order beliefs

- Modified level-k reasoning framework is needed to account for these issues